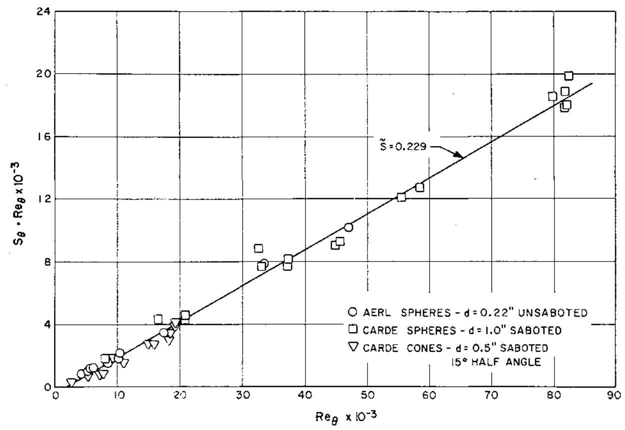

Fig. 3 Strouhal numbers for the hypersonic wake ($M_\infty \approx$ 14), spheres and cones; based on total wake-momentum thickness $\theta = (C_D A/2\pi)^{1/2}$ as characteristic length.

visible by the wake's own luminosity. As the model and its wake move down the ballistic range their image moves transverse to the motion of the film producing on the film an x-t diagram of the flow field events. Figure 1 is a typical example of a drum camera streak photograph. The pellet is moving from left to right and traces the bright straight line; the "streaks" are the gross traces of the large scale eddies as they fall behind the body and slow down. The hypersonic Strouhal number is obtained by eye by counting the number of prominences along a line of the film parallel to the trace of the pellet. A counting line is chosen as near to the body as the streak structure is resolvable, i.e., immediately downstream of the base resolution limit of the drum camera technique. The following rules are observed: 1) each prominence is counted and is associated with one cycle of the periodic phenomenon, and 2) fluctuations within an eddy or with a wavelength less than a body radius are ignored. If the projectile moves a real distance x in the time during which N events are counted, then the Strouhal number is NL/x.

A shoulder Reynolds number (flow direction parallel to freestream) is computed as being representative of the separated flow conditions in the near wake where the measurement is made. The best estimates of the flow chemistry indicate an equilibrium flow across the sphere bow shock for the ballistic range conditions of these runs. As in Fay and Goldburg, the formula for the shoulder Reynolds number of spheres in air is taken approximately as

$$R = 1780[V(\text{kft/sec})]^{0.6}[\rho(\text{cm})][L(\text{cm})] \qquad (3)$$

A shoulder Reynolds number for cones is computed by taking the flow through the shock wave and the Prandtl-Meyer expansion of the cone angle using the conical flow tables of Romig[3] and NACA 1135.[4]

Results

Rayleigh's empirical relationship between the Strouhal and Reynolds number

$$S = \tilde{S}\,[1 - (Re_T/Re)]$$

(where $\tilde{S}$ is the asymptotic value of the Strouhal number for large Re, and Re_T is the Reynolds number for which S goes to zero) yields a straight line when $S \times Re$ is plotted against Re. This line when extended to $S = 0$ will yield a transition Reynolds number Re_T.

Figure 2 is such a plot of $S \times Re$ vs Re based on body diameter as the characteristic length. Indicated on this plot are two lines

$$S_d = 0.66(1 - 3180/Re_d) \qquad \text{spheres} \qquad (4)$$

$$S_d = 1.72(1 - 18100/Re_d) \qquad \text{cones} \qquad (5)$$

Each line is the least-square fitted line passing through the respective data. Figure 3 shows the same data plotted using θ as the characteristic length. Based on the preceding arguments, as in the incompressible case, we expect that θ may be the correlating length parameter allowing a single line to be drawn through both the sphere and cone points. In the plot is shown the least-square fitted line

$$S_\theta = 0.229[1 - (1820/Re_\theta)] \qquad (6)$$

Finally, since the frequency measurements are based on highlights of luminosity in the hypersonic wake, the question is raised whether the apparent periodicity is caused by unsymmetrical heating of the projectile in the gun barrel. The answer is apparently not. Calculations show that the mechanical integrity of the bodies would be jeopardized at the kilocycle frequencies involved; in addition, the spheres shot at the AERL range were unsaboted whereas those at CARDE were saboted and thus protected from gun-barrel heating.

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Forced Vibrations of a Burning Rocket

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Introduction

THE loss of mass and stiffness of a burning cylindrical grain affects the dynamic response of a solid-propellant rocket. The viscoelastic behavior of the solid-propellant material is another factor that influences the forced vibrations of a burning rocket. In the present note the effects of ablation and viscoelastic damping are considered in a study of the dynamic response of an encased viscoelastic cylinder with an ablating inner surface.

A time-dependent pressure is applied at the ablating inner surface of the cylinder. The cylinder material is viscoelastic in shear, and it is assumed incompressible in bulk. As a consequence of the incompressibility assumption the dilatational wave velocity is infinite, and a forced vibration is immediately started without initial wave effects. Solid-propellant materials show very high bulk moduli, and they are often considered as incompressible.

Special attention has been devoted to the circumferential stress at the ablating inner surface. The analysis is valid for arbitrary ablation rates. The analogous problem of the encased elastic cylinder has been discussed in an earlier note.[1]

Statement of the Problem

A long viscoelastic cylinder is considered with a circular port of monotonically increasing radius $a(t)$ and a constant

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outer radius b at which the cylinder is bonded to a thin elastic shell. Both the cylinder and the surrounding shell are in a state of plane strain. The cylinder-shell combination is subjected to a time-dependent pressure, applied at the ablating inner surface.

Let s_{ij} and e_{ij}, respectively, denote the components of deviatoric stress and strain. If the body is at rest prior to $t = 0$, the stress-strain relation for the incompressible material of the cylinder can be expressed in the form

$$s_{ij}(r, t) = 2\int_{0^-}^{t} G(t - s)de_{ij}(r, s) \quad (1)$$

in which $G(t)$ is the relaxation function in shear. For axisymmetric deformation of an incompressible cylinder, the radial displacement is of the form

$$u(r, t) = k(t)/r \quad (2)$$

We can then derive from Eq. (1)

$$\sigma_r - \sigma_\theta = 2\int_{0^-}^{t} G(t - s)d(\epsilon_r - \epsilon_\theta) = -(4/r^2)\int_{0^-}^{t} G(t - s)dk(s) \quad (3)$$

Analogously to the problem of the encased elastic core,[1] the expressions for $u(r, t)$ and $\sigma_r - \sigma_\theta$, respectively Eq. (2) and Eq. (3), are substituted in the equation of motion, and the resulting equation for σ_r is subsequently integrated with respect to r. The thus obtained expression for σ_r is subject to boundary conditions at $r = a(t)$ and $r = b$. At the ablating inner surface $r = a(t)$, the radial stress is prescribed as

$$\sigma_r[a(t), t] = -\sigma_i(t) \quad (4)$$

The radial stress at $r = b$ is derived from the equation of motion of an element of the thin surrounding shell.[1] By substituting the boundary conditions in σ_r, the following equation for $k(t)$ is obtained:

$$\left\{\rho_p \ln\left[\frac{b}{a(t)}\right] + \rho_s\left(\frac{h}{b}\right)\right\}\ddot{k}(t) + \left(\frac{h}{b}\right)\left(\frac{E^*}{b^2}\right)k(t) + 2\left[\frac{1}{a(t)^2} - \frac{1}{b^2}\right]\int_{0^-}^{t} G(t - s)dk(s) = \sigma_i(t) \quad (5)$$

In Eq. (5) $E^* = E_s(1 - \nu_s^2)$, and ν_s is Poisson's ratio. A subscript s denotes a material constant of the shell. The thickness of the case is denoted by h. The solution of the integrodifferential equation, Eq. (5), is subject to the initial conditions

$$k(t) = \dot{k}(t) = 0 \qquad \text{for} \qquad t \leq 0 \quad (6)$$

Removing the discontinuity at $t = 0$ and integrating by parts, Eq. (5) is rewritten as

$$\left\{\rho_p \ln\left[\frac{b}{a(t)}\right] + \rho_s\left(\frac{h}{b}\right)\right\}\ddot{k}(t) + \left\{\left(\frac{h}{b}\right)\left(\frac{E^*}{b^2}\right) + 2G(0)\left[\frac{1}{a(t)^2} - \frac{1}{b^2}\right]\right\}k(t) + 2G(0) \times \left[\frac{1}{a(t)^2} - \frac{1}{b^2}\right]\int_{0^+}^{t} g'(t - s)k(s)ds = \sigma_i(t) \quad (7)$$

where the prime denotes differentiation with respect to the argument, and

$$G(t) = G(0)g(t) \quad (8)$$

For convenience the following dimensionless quantities are introduced:

$$\tau = t/t_f \qquad a(\tau) = a_0\alpha(\tau) \qquad K(\tau) = k(\tau)/b^2 \quad (9)$$

In Eq. (9) t_f is the total burning time. The ablation function $\alpha(\tau)$ runs from $\alpha(0) = 1$ to $\alpha(1) = b/a_0$. Introduction of the dimensionless quantities into Eq. (7) yields

$$\ddot{K}(\tau) + q^2\chi(\tau)K(\tau) + q^2\psi(\tau)\int_{0^+}^{\tau} K(s)g'(\tau - s)ds = q^2\varphi(\tau) \quad (10)$$

In Eq. (10)

$$\chi(\tau) = \{(h/b) + 2[G(0)/E^*] \times [(b/a_0)^2/\alpha(\tau)^2 - 1]\}/\gamma(\tau) \quad (11)$$

$$\psi(\tau) = 2[G(0)/E^*][(b/a_0)^2/\alpha(\tau)^2 - 1]/\gamma(\tau) \quad (12)$$

$$\varphi(\tau) = [\sigma_i(\tau)/E^*]/\gamma(\tau) \quad (13)$$

$$\gamma(\tau) = (h/b) + (\rho_p/\rho_s)\ln[(b/a_0)/\alpha(\tau)] \quad (14)$$

and q is the dimensionless burning time

$$q = t_f(E^*/\rho_s)^{1/2}/b \quad (15)$$

Inspection of Eqs. (10–15) reveals that the parameters in the present problem are the structural parameters (h/b) and (b/a_0), the material parameters (ρ_p/ρ_s), $G(0)/E^*$, and $g'(\tau)$, the ablation function $\alpha(\tau)$, and the burning time q.

It is not without profit to investigate the orders of magnitude of the various parameters for a typical situation. We choose

$$h/b = 1/200 \qquad \rho_p/\rho_s = 1/5 \qquad b/a_0 = 3$$
$$b = 20'' \qquad t_f = 60 \text{ sec} \quad (16)$$

The parameter q is large; for a steel case it is found

$$q = 6.10^5 \quad (17)$$

The material of the cylinder is soft as compared to that of the case. A reasonable proportionality is

$$G(0)/E^* = 10^{-4} \quad (18)$$

With Eqs. (11, 12, and 14) it is then seen that

$$(\rho_s h/\rho_p b) \leq \chi(\tau) \leq 1 \quad (19)$$

In the next section it is discussed that the magnitudes of the slopes $g''(\tau)$, $g'''(\tau)$, etc. limit the analysis. The allowable values of $g''(\tau)$ are determined from the requirement

$$A(\tau) = |g''(\tau)/q\chi(\tau)^{1/2}g'(\tau)| \ll 1 \quad (20)$$

For many viscoelastic materials $[g'(\tau)]^2 \sim |g''(\tau)|$, and Eq. (20) becomes

$$|g'(\tau)/q\chi(\tau)^{1/2}| \ll 1 \quad (21)$$

To obtain a highest admissible value for $dG(t)/dt$, we set the right-hand side of Eq. (21) equal to 0.01. With Eqs. (8, 16–18),

$$|[dG(t)/dt]/G(0)| < (0.01)q\chi^{1/2}/t_f \simeq 20 \quad (22)$$

Method of Solution

If $g'(\tau) \equiv 0$ the integrodifferential equation (10) reduces to an ordinary differential equation, which governs the analogous elastic problem. A solution of the elastic problem was obtained[1] by taking advantage of the large magnitude of the real parameter q. It was shown that for a suddenly applied pressure and excluding terms of order $(1/q)$ the elastic solution consists of the quasi-static solution plus a dynamic deviation. Because the elastic solution is a limit case of the viscoelastic solution, we feel justified in seeking a dynamic viscoelastic solution of the form

$$K(\tau) = K(\tau)_{qs} + D(\tau) \quad (23)$$

In Eq. (23), $K(\tau)_{qs}$ is the quasi-static solution that satisfies the equation

$$\chi(\tau)K(\tau)_{qs} + \psi(\tau)\int_{0^+}^{\tau} K(s)_{qs}g'(\tau - s)ds = \varphi(\tau) \quad (24)$$

Various numerical methods and approximate analytical methods are available to solve Eq. (24). Substituting Eq. (23) into (10), we now obtain for $D(\tau)$

$$\ddot{D}(\tau) + q^2\chi(\tau)D(\tau) + q^2\psi(\tau)\int_{0^+}^{\tau} D(s)g'(\tau - s)ds = -\ddot{K}(\tau)_{qs} \quad (25)$$

Extending Horn's method,[2] a solution of Eq. (25) is sought of the form

$$D(\tau) = \{m(\tau)\cos[q\omega(\tau)] + [n(\tau)/q]\sin[q\omega(\tau)]\}f(\tau, q) \quad (26)$$

where

$$f(\tau, q) = 1 + \sum_{j=1}^{\infty} \frac{f_j(\tau)}{q^j} \quad (27)$$

In the first approximation all terms that contain q in the denominator are neglected

$$D(\tau) = m(\tau)\cos[q\omega(\tau)] \quad (28)$$

The assumed solution Eq. (28) is substituted into the integro-differential equation (25). Since q is a large real parameter the integral can be evaluated by the method of stationary phase[2]

$$\int_{0^+}^{\tau} m(s)g'(\tau - s)\cos[q\omega(s)]ds = \left\{1 + 0\left[\frac{g''(\tau)}{g'(\tau)q\dot{\omega}(\tau)}\right]\right\}\frac{m(\tau)g'(0)}{q\dot{\omega}(\tau)}\sin[q\omega(\tau)] \quad (29)$$

The assumption that the second term inside the accolade can be neglected relative to unity leads to the requirement of Eq. (20). Substituting Eq. (29) into (25) and collecting terms in q^2 and q, we obtain

$$-\dot{\omega}^2 + \chi(\tau) = 0 \quad (30)$$

$$2\dot{m}\dot{\omega} + \ddot{\omega}m - g'(0)\psi(\tau)m/\dot{\omega} = 0 \quad (31)$$

Solving Eqs. (30) and (31) and taking into account that $D(0^+) = -K(0^+)_{qs}$, the approximate solution for $D(\tau)$ is found as

$$D(\tau) = \frac{-\varphi(0^+)}{\chi(0)^{3/4}\chi(\tau)^{1/4}}\exp\left[\frac{g'(0)}{2}\int_0^{\tau}\frac{\psi(s)}{\chi(s)}ds\right] \times \cos\left[q\int_0^{\tau}\chi(s)^{1/2}ds\right] + 0[A(\tau)] \quad (32)$$

where $A(\tau)$ is defined by Eq. (20).

It is seen from Eqs. (24) and (32) that the solution $D(\tau)$ does not satisfy the initial condition $\dot{K}(0) = 0$. In order to satisfy the initial condition on $\dot{K}(\tau)$, terms of order $(1/q)$ have to be included and the more general expression, Eq. (26), must be used. It is of interest that the frequency of the vibration increases with time, since $\chi(s)$ is a positive function. In the present analysis it is assumed that $\sigma_i(0^+) \neq 0$, i.e., the internal pressure is applied as a step function. Higher-order terms in $(1/q)$ have to be taken into account if the internal pressure increases gradually. In the latter case the solution is of order $[\dot{\sigma}_i(\tau)/q]$. The dynamic part of the solution is then negligible if the rise time of the applied pressure is much larger than the natural frequency, i.e., if $\dot{\sigma}_i(\tau)/q \ll 1$.

Circumferential Stresses

With Eqs. (3, 4, and 32) the circumferential stress at the ablating inner surface can be expressed as

$$\sigma_\theta[a(\tau), \tau] = \sigma_\theta]_{qs} + \sigma_\theta]_{do} \quad (33)$$

In Eq. (33), $\sigma_\theta]_{qs}$ is the quasi-static solution and $\sigma_\theta]_{do}$ is the dynamic overstress:

$$\sigma_\theta]_{do} = 4(b/a_0)^2\alpha(\tau)^{-2}G(0)[D(\tau) + \int_{0^+}^{\tau} g'(\tau - s)D(s)ds] \quad (34)$$

The integral in Eq. (34) is again evaluated by the method of stationary phase:

$$\sigma_\theta]_{do} = S(\tau)\left\{\cos\left[q\int_0^{\tau}\chi(s)^{1/2}ds\right] + [g'(0)\chi(\tau)^{-1/2}/q]\sin\left[q\int_0^{\tau}\chi(s)^{1/2}ds\right]\right\} \quad (35)$$

in which

$$S(\tau) = \frac{M\sigma_i(0^+)}{\alpha(\tau)^2}\left[\frac{\chi(0)}{\chi(\tau)}\right]^{1/4}\exp\left[\frac{g'(0)}{2}\int_0^{\tau}\frac{\psi(s)}{\chi(s)}ds\right] \quad (36)$$

where

$$M = 4(b/a_0)^2/[(h/b)E^*/G(0) - 2 + 2(b/a_0)^2] \quad (37)$$

From Eq. (35) the amplitude of the dynamic circumferential overstress is found as

$$\sigma_\theta^*(\tau) = S(\tau)\{1 + [g'(0)/q]^2\chi(\tau)^{-1}\}^{1/2} \quad (38)$$

The solution Eq. (35) shows that for a suddenly applied pressure, and excluding terms of order $A(\tau)$ Eq. (20), the slope of the relaxation function at $\tau = 0$ principally governs the damping of the dynamic response. The slope $g'(0)$ is usually of large magnitude, and the dynamic overstress is therefore damped out in a short time. If damping takes place that rapidly, it is not to be expected that the influence of ablation is as pronounced as for the elastic core. The ablation function and the complete relaxation function do, of course, severely influence the quasi-static solution.

In Fig. 1, $\sigma_\theta^*(\tau)$ is shown for numerical values as defined by Eqs. (16) and (17). Two ablation functions are considered

$$\alpha(\tau) = (1 - \kappa\tau)^{-1/2} \quad (39)$$

where $\kappa = 1 - (a_0/b)^2$.

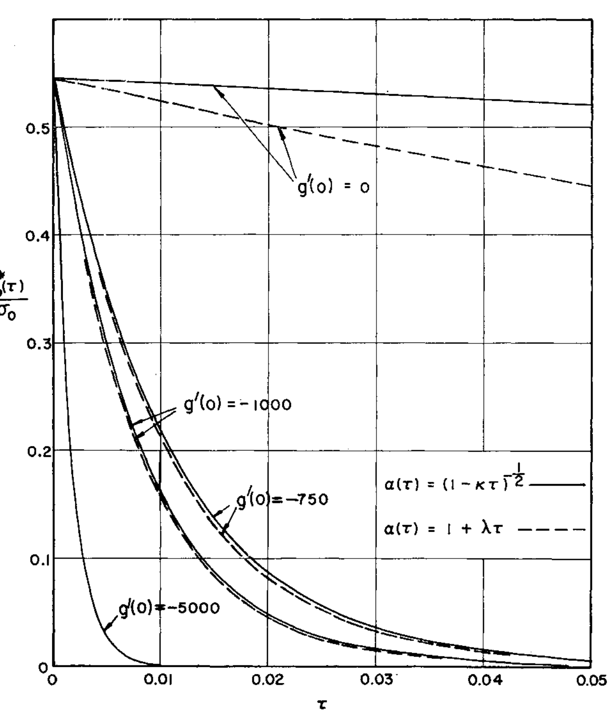

Fig. 1 Amplitude of the circumferential stress at the ablating inner surface.

and the linear function

$$\alpha(\tau) = 1 + \lambda\tau \tag{40}$$

where $\lambda = (b/a_0) - 1$. The viscoelastic solution is compared to the elastic solution for an ablating cylinder with shear modulus $G(0)$.

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A Self-Calibrating Probe for Measuring Atom Concentration in a Hypersonic Flow

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Nomenclature

U = flow velocity
ρ = flow density
μ = viscosity
T = temperature
j = 0 for two-dimensional flow, 1 for three-dimensional flow
Sc = Schmidt number
Pr = Prandtl number
Le = Lewis number
$h_R{}^\circ$ = heat of dissociation
α = atom concentration
H = total enthalpy
ϕ_c = catalytic efficiency
d = probe diameter

Subscripts

c = catalytic
nc = noncatalytic
e = edge of boundary layer
w = wall conditions
∞ = freestream conditions
2 = two-dimensional flow
3 = three-dimensional flow

Introduction

In hypersonic shock tunnels, high-pressure, high-enthalpy and highly dissociated gases are expanded in nozzles having large area ratios to get hypersonic Mach numbers. The expansion process gives rise to nonequilibrium effects, which make it difficult to obtain accurate measurements of flow quantities such as U, ρ, α, and T. Attempts have been made to predict the state of the flow along the nozzle centerline by measuring static pressure. However, since the static pressure is not as sensitive to nonequilibrium flow effects as atom concentration, an independent measurement of this quantity would throw more light on this phenomenon. A knowledge of the atom concentration is also essential for predicting the flow variables of the freestream in the nozzle.

The principle[1] is to arrange the experimental conditions so that the shock layer and boundary layer around the probe are frozen. If a probe containing catalytic and noncatalytic heat-transfer gages (herein to be called a differential heat-transfer gage) mounted side by side is placed in a dissociated hypersonic stream, then the freestream atom concentration can be determined from the measured values of heat transfer to the differential heat-transfer gage and the stagnation-point heat-transfer relation[2]:

$$(q_{c-nc}) = 2^{j/2} \times 0.54 S_c{}^{-0.63}(H_e - H_w) \times (\beta\mu_e\rho_e)^{1/2}(h_R{}^\circ \alpha_\infty/H_e)\phi_c \tag{1}$$

where

$$\phi_c = 1/[1 + (S/k_w)]$$
$$S = 0.54 \times 2^{j/2}(\beta\mu_e\rho_e)^{1/2}S_c{}^{-0.63}\rho_w{}^{-1}$$
$$\beta = (2U_\infty/d)\{(\rho_\infty/\rho_e)[2 - (\rho_\infty/\rho_e)]\}^{1/2}$$

However, from Eq. (1) it is apparent that flow quantities like ρ_e, μ_e, and α_∞, etc. have to be known to obtain α_∞. Also, the catalytic efficiency ϕ_c of the silver-coated gage has to be determined independently. At present, there have been a few attempts[3,4] to measure the gage catalytic efficiency at very low speeds (50 to 100 fps). However, these experiments do not simulate the actual conditions that the catalytic gage is subjected to when placed in a hypersonic stream. The catalytic efficiency (ϕ_c) of the gage has to be measured in the actual environment in which the gage operates when it is used to obtain the freestream atom concentration. In addition, there is an uncertainty in the value of viscosity[5] of a dissociated gas at high temperature.

In order to avoid the previously mentioned difficulties in measuring atom concentration, the following method may be adopted so that the probe is self-calibrating with respect to its catalytic efficiency. Furthermore, by using this method, there is no need to know the viscosity, density, and flow velocity.

1. Theoretical Considerations

Equation (1) gives the general expression for differential heat transfer. Therefore, if two differential heat-transfer gage models (one three-dimensional and the other two-dimensional) are mounted side by side in a hypersonic stream and the differential heat transfer to both models is measured simultaneously, then the expressions for heat transfer are given by 1) three-dimensional flow

$$(q_{c-nc})_3 = 0.763 S_c{}^{-0.63}(H_e - H_w)(\beta_3\mu_e\rho_e)^{1/2}(h_R{}^\circ\alpha_\infty/H_e)\phi_{c3} \tag{2}$$

where

$$\phi_{c3} = 1/[1 + (S_3/K_w)] \qquad S_3 = 0.763(\beta_3\mu_e\rho_e)^{1/2}S_c{}^{-0.63}\rho_w{}^{-1}$$
$$\beta_3 = (2U_\infty/d_3)\{(\rho_\infty/\rho_e)[2 - (\rho_\infty/\rho_e)]\}^{1/2}$$

and 2) two-dimensional flow

$$(q_{c-nc})_2 = 0.54 S_c{}^{-0.63}(H_e - H_w)(\beta_2\mu_e\rho_e)^{1/2}(h_R{}^\circ\alpha_\infty/H_e)\phi_{c2} \tag{3}$$

where

$$\phi_{c2} = 1/[1 + (S_2/K_w)] \qquad S_2 = 0.54(\beta_2\mu_e\rho_e)^{1/2}S_c{}^{-0.63}\rho_w{}^{-1}$$
$$\beta_2 = (2U_\infty/d_2)\{\rho_\infty/\rho_e[2 - (\rho_\infty/\rho_e)]\}^{1/2}$$

In the preceding equations the velocity of the wall chemical reaction of the catalytic surface (k_w) is assumed to be the same for both gages. Also, the flow variables U, ρ, and α are assumed uniform in the core of the flow in which the two models are mounted.

Dividing Eq. (2) by Eq. (3) yields

$$(q_{c-nc})_3/(q_{c-nc})_2 \equiv \delta_{32} = (2\beta_3/\beta_2)^{1/2}(\phi_{c3}/\phi_{c2}) \tag{4}$$

S_2, S_3, and ϕ_{c2}, ϕ_{c3} are interrelated and can be expressed as

$$S_3/S_2 = (2\beta_3/\beta_2)^{1/2} \equiv (2d_2/d_3)^{1/2} \tag{5}$$

$$\phi_{c3}/\phi_{c2} = \phi_{c3} + (1 - \phi_{c3})(d_3/2d_2)^{1/2} \tag{6}$$

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